OPTICALLY PUMPED IMAGE LIGHT AMPLIFICATION

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FOREWORD

This report was prepared by Electro-Optical Systems, Inc., Pasadena, California, in fulfillment of Air Force Contract AF 33(657)-11326 for the Headquarters, Aeronautical Systems Division, Air Force Systems Command, United States Air Force. The work was conducted under Project and Task Number 4156-05. Dr. B. Kazan is the Project Supervisor: Telephone Number Area Code 213, 449-1230. The secondary report numbers assigned by Electro-Optical Systems, Inc. is 3990-Q-1.

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ABSTRACT

Work during the first quarter was concerned largely with consideration of some of the basic problems associated with a stimulated emission intensifier. Primary attention was given to the problem of the generation of background light caused by spontaneous emission and its amplification in passing through the intensifier. Because of the high fluorescence level occurring, schemes for possibly lowering this background were considered. One of these involves the incorporation of a non-uniform pumping intensity or the incorporation of a non-uniform density of luminescent states along the amplifier axis. Another possibility considered is the use of an auxiliary absorbing medium placed between the intensifier output and the observer's eye, which would be optically non-linear in such a way that its light transmission is low below a specified light level, but high above this threshold. The factors concerned with the choice of a material for the amplifier itself were separately considered, and the problem of obtaining uniform pumping in the case of an optically-pumped amplifier was also considered. To study the problems which might arise because of image deterioration in passing through a stimulated-emission intensifier, preliminary experimental tests were initiated. In view of the difficult problems anticipated in the development of an intensifier based on stimulated emission, an analysis has also been initiated on another possible form of intensifier which may have greater bandwidth and lower fluorescence background characteristics.

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ILLUSTRATIONS

1. INTRODUCTION

In accordance with the requirements of the contract the purpose of the following work program is to investigate the feasibility of a low-level image amplifier employing "stimulated emission." Although a variety of vacuum and solid-state types of image intensifiers have been developed over the past number of years, such devices all involve some form of conversion process such as from optical to electrical energy or vice versa. In other words, although an intensified pattern of bright and dark areas is generated at the output of the intensifier the observer views the output image as an optical source originating at the output surface of the intensifier. Since it is necessary with such intensifiers to optically image the external scene onto the input surface of the intensifier, the depth of focus of the input image is limited by the optics employed. Although, to some degree depth perception can be achieved with a binocular arrangement of intensifiers, in order for the observer to fully scan the scene in depth it is necessary to vary the input optical focusing.

In the proposed scheme to be investigated, the approach is to obtain intensification without the formation of a real optical image on an absorbing surface. Instead, the individual light rays are to be intensified by a process such as stimulated emission. The intensifier would thus consist, for example, of a block of material through which the observer would view the external scene as he would in the absence of the intensifier, focusing and directing his eyes as he chose on scene details at different distances, but seeing the scene in much brighter form.

Among the many problems which emerge in considering such an intensifier are those concerned with the very high fluorescence background of a stimulated emission amplifier, its inherently narrow bandwidth, the pumping power required, the difficulty of providing uniform pumping and limitations on the angle of view. Some of these problems are discussed in the following sections.

2. NOISE AND FLUORESCENCE PROBLEMS

2.1 Basic Noise Processes

Perhaps the most fundamental problem associated with the development of a stimulative-emission image intensifier is the occurrence of spontaneous random photon emission which produces a noise of fluorescent background throughout the body of the amplifier. This emission is also amplified by the laser medium, and as shown below, requires a relatively high input signal if it is to be observed above fluorescent output. In the discussion below two somewhat different approaches are taken on the problem of estimating the noise, both of these approaches, however, leading to approximately the same magnitude of noise.

In one approach a small filament of active material is considered (see Fig. 1). It is assumed that this filament is of the order of the wavelength of the input signal light, i.e., it is propagating energy in its fundamental mode only. The increase in signal dP_s, resulting from traversing a length dx of this filament is:

$$dP_s = J(N_2 - N_1)\ P_s dx(\text{watts/cps}) \tag{3.1}$$

where J is a constant proportional to B, the coefficient of stimulated emission and N_2 and N_1 are the populations of the upper and lower states respectively. The increase in noise power, dP_n, is given by

$$dP_n = J(N_2 - N_1)\ P_n dx + KN_2 dx(\text{watts/cps}) \tag{3.2}$$

where K is a constant proportional to A, the coefficient of spontaneous emission.

The constant K may be evaluated by noting that in thermal

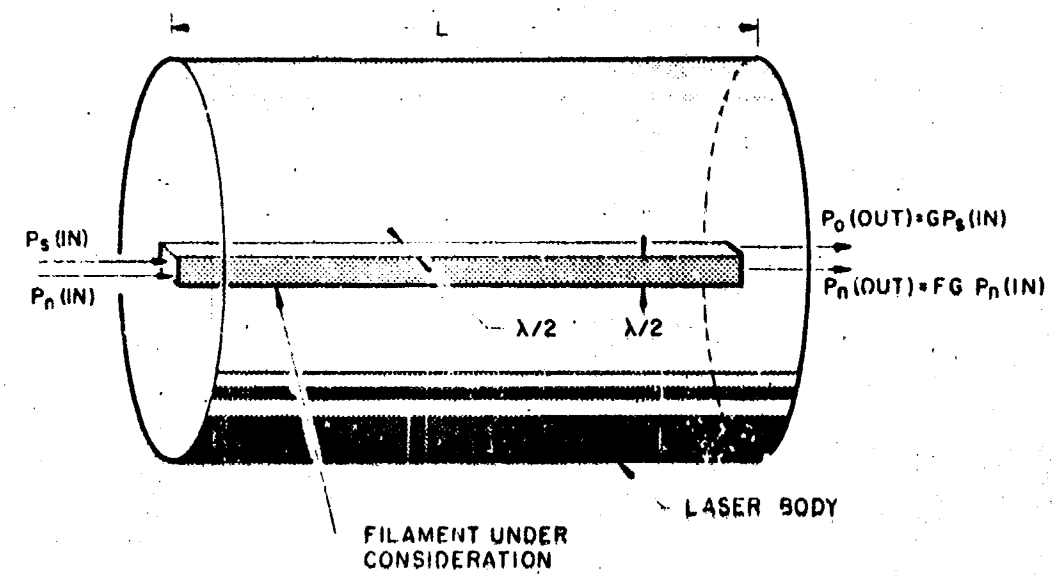


Figure 1 Noise and Power Relations in a Filamentary Laser Element

equilibrium $dP_n = 0$ so that

$$(N_{20} - N_{10})\, P_{no} + KN_{20} = 0 \tag{3.3}$$

where N_{10} and N_{20} are the thermal equilibrium values of N_1 and N_2.

Thus $$K = J\left(\frac{N_{10}}{N_{20}} - 1\right) P_{n0} \tag{3.4}$$

In thermal equilibrium the ratio $\frac{N_{10}}{N_{20}}$ is given by the Boltzmann factor $e^{h\nu/kT}$ while the noise power in a single degree of freedom of the system at temperature T is $h\nu/(e^{h\nu/kT}-1)$ so that

$$K = Jh\nu \tag{3.5}$$

Substituting this value into Eq. (3.2) gives

$$dP_n = J\,(N_2 - N_1)\, P_n dx + Jh\nu N_2 dx \tag{3.6}$$

Integrating (1) for a filament of length L gives

$$\frac{P_s\ (out)}{P_s\ (in)} = e^{J(N_2 - N_1)L} = G \tag{3.7}$$

where G is the power gain.

Integrating Eq. (3.6) gives

$$P_n\ (out) = GP_n\ (in) + (G-1)\ h\nu \left(\frac{N_1}{N_2 - N_1}\right) \tag{3.8}$$

This expression shows the output noise to consist of two components:

1. An amplified version of the input noise GP_n (in) and
2. The internally generated noise $(G-1)\ h\nu\left(\frac{N_2}{N_2 - N_1}\right)$.

In a high gain ($G \gg 1$) laser which is pumped so that $N_2 \gg N_1$ the internally generated noise referred to the input is $h\nu$ watts/cps. Since each photon has an energy of $h\nu$, this noise power corresponds to an average photon flux of one per second.

This computation has been carried out for a single degree of freedom of the system corresponding to a wave guide or an optical fibre operated in its fundamental mode, so that this noise power is emitted from an area of about $(\lambda/2)^2$ cm^2. Therefore, the average equivalent input noise power per unit area is given approximately by

$$P_n' \text{ (equiv)} \approx \frac{h\nu}{(\lambda/2)^2} = \frac{4h\nu^3}{c^2} \text{ (watt/cps/cm}^2\text{)} \qquad (3.9)$$

Comparing this intensity of radiation with the emission from a black body,

$$\frac{2\pi h\nu^3}{c^2} (e^{h\nu/kT}-1)^{-1} \text{ (watt/cps/cm}^2\text{)} \qquad (3.10)$$

shows that the two are approximately equal when

$$h\nu = kT$$

or

$$T = h\nu/k$$
$$= 1.5 \times 10^4/\lambda$$

where λ is measured in microns. For a laser operating in the visible (λ = 0.6 micron) the noise equivalent black body temperature would be 2.5×10^4 °K. In other words, if a laser amplifier were used to view a black body, the temperature of that body would have to be 2.5×10^4 °K in order to achieve unit signal to noise ratio.

The noise equivalent input power from Eq. (3.9) would be 1.9×10^{-10} watts/cm^2/cps. For a laser bandwidth of 2.4×10^{11} cps (a measured value for Eu^{3+} in Y_2O_3) the power density would be 47 watts/cm^2. Thus for an object viewed through this type of laser to appear as bright as the background emission, it would have to radiate 80 watts/cm^2 within the passband of the device.

Another approach for estimating the background noise is as follows: Consider a distant object being imaged by the eye through

a block of the amplifying substance. The eye will see an image of amplified brightness

$$B_a = B_o e^{\alpha \ell} \quad (3.11)$$

where B_o is the unamplified object brightness

α is the amplification coefficient of the substance

ℓ is the optical path length through the substance.

The amplification coefficient α may be related to the coefficient of spontaneous emission B as follows: The rate at which stimulated emission takes place in the amplifying substance is given by the formula

$$\frac{dN_2}{dt} = UB\ (N_2 - N_1) \qquad \left(\frac{1}{cm^3}\right) \quad (3.12)$$

where U = density of radiation (joule/cm^3)

B = probability per unit time that an atom will be stimulated to emit a photon by the presence of unit radiation density

N_2 and N_1 = the populations of the upper and lower states respectively.

In a slice of thickness dx the increase in power, dP, due to stimulated emission for a wave front traveling along the x direction is given by

$$dP = h\nu \frac{dN_2}{dt} dx \qquad (\text{watts/cm}^2) \quad (3.13)$$

where $h\nu$ is the photon energy.

Substituting for $\frac{dN_2}{dt}$ gives

$$dP = h\nu UB(N_2 - N_1)dx \qquad (\text{watts/cm}^2) \quad (3.14)$$

However, the excited atoms in the amplifying substance will also emit photons of frequency ν spontaneously with a probability A

per unit time, and a unit volume of the material will radiate an amount of power P given by

$$P = \frac{h\nu\, A\, N}{4\pi} \qquad \text{(watt/ster.-cm}^3\text{)} \qquad (3.15)$$

The power per steradian radiated through one face of the amplifying slab is found by integrating the power emitted by the volume element dv multiplied by the gain due to the path length. Hence the brightness B_s due to spontaneous emission is given by

$$B_s = \int_{x=o}^{\ell} \epsilon^{\alpha x} P dv = \frac{P}{\alpha}\left(\epsilon^{\alpha\ell} - 1\right) \qquad \text{(watt/ster.-cm}^2\text{)} \qquad (3.16)$$

The necessary unamplified object brightness B_o which will yield an image of brightness B_a just equal to the background brightness B_s due to spontaneous emission is found by setting $B_a = B_s$ giving

$$B_o = \frac{P}{\alpha} \frac{\epsilon^{\alpha\ell} - 1}{\epsilon^{\alpha\ell}} \qquad \text{(watt/ster.-cm}^2\text{)} \qquad (3.17)$$

If the gain $\epsilon^{\alpha\ell} \gg 1$ then the brightness B_o is given by

$$B_o = \frac{P}{\alpha} = \frac{A_c}{4\pi B} \frac{N_2}{N_2 - N_1} \qquad \text{(watt/ster.-cm}^2\text{)} \qquad (3.18)$$

From fundamental considerations (Ref. 8) it can be shown that

$$\frac{A}{B} = 8\pi h\nu^3/c3$$

so that

$$B_o = \frac{2h\nu^3}{c^2} \frac{N_2}{N_2 - N_1} \qquad \text{(watts/ster.-cm}^2\text{-cps)} \qquad (3.19)$$

In an efficiently pumped, three-level system, it should be possible to attain nearly complete population inversion, (i.e. $N_2 \gg N_1$) so that the ratio $\frac{N_2}{N_2 - N_1}$ approaches unity. The formula for the

required brightness, B_o, then becomes

$$B_o = 2h\nu^3/c^2 \qquad \text{(watts/ster.-cm}^2\text{-cps)} \qquad (3.20)$$

If the device is to operate in the visible where $\lambda = c/\nu = .6$ micron, then the required scene brightness is given by:

$$B_o = 1.8 \times 10^{-10} \qquad \text{(watts/ster.-cm}^2\text{-cps)}$$

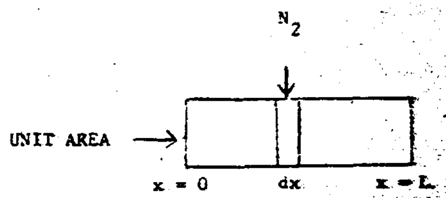
If the bandwidth of the amplifier is 2.4×10^{11} cps (the measured value for the fluorescence of the Eu^{3+} level in yttrium oxide) the calculated value of scene brightness to produce unity signal-to-background ratio is 44 watts/ster. -cm^2. Assuming a Lambertian distribution of radiation, the power radiated per square centimeter would be $\pi \times 44$ watts/cm^2.

3.2 Effects of Non-Uniform Doping or Pumping

The results deduced in Section 3.1 for the fluorescence or noise output were for an amplifier with uniform gain per unit length. A question which arises is, whether an improvement in signal-to-noise ratio could be made by varying the gain per unit length in an appropriate manner. A possible arrangement is suggested by analogy with a low noise, low gain preamplifier ahead of a high gain amplifier in more conventional vacuum tube circuits. In a stimulated-emission amplifier variation in the gain per unit length may be achieved by variation of the density N_2 of excited states. This may be accomplished either by variation of the impurity doping or by variation of the intensity of pumping. The density of excited states, in general, is proportional to the product of active impurity density and the pumping intensity.

Consider a four-level, stimulated emission amplifier in which the density of atoms in the upper level of the laser transition is N_2 (x), where x is distance from the input end of the amplifier.

The density, $N_1(x)$, of the terminal laser state will be assumed negligible since this produces the highest gain and the lowest noise. The overall gain of the amplifier for a photon starting at point x and traveling in the x direction



$$g(x) = \exp\left(\int_x^L \alpha(x')dx'\right) \tag{3.21}$$

where $\alpha(x)$ is the absorption or emission constant for the transition and is directly proportional to $N_2(x)$ since N_1 is negligible. Thus $\alpha = K_1N_2$ where K_1 is a constant characteristic of the transition. If $\alpha(x)$ is the absorption coefficient, K_1 is negative. The overall gain, G, of the amplifier may be written as

$$G = g(L) = \exp\left(\int_0^L \alpha(x')dx'\right) \tag{3.22}$$

Likewise note $g(0) = 1$ since both limits of the integral are identical.

The noise, or fluorescent power, originating in an elemental volume, dx, is given simply by $K_2N_2(x)dx$ where K_2 is another constant. The fluorescent power, dn, at the end, L, which originates at dx but is amplified through the medium, is given by

$$dn(x') = K_2N_2(x')f(x')dx', \tag{3.23}$$

and the total noise at the output end due to fluorescence all along the material is the sum of all elemental volumes.

$$n = \int_0^L K_2 N_2 (x') g(x') dx' \tag{3.24}$$

Substituting $\mu(x) = K_1 N_2(x)$, this equation becomes

$$n = \frac{K_2}{K_1} \int g(x') \mu(x') dx' \tag{3.25}$$

Integrating this by parts using $\mu(x)dx$ as the differential, using Eq.(3.21) and the known integral

$$\int \ln g \, dg = g \ln g - g \tag{3.26}$$

n becomes,

$$\frac{K_1}{K_2} n = g \Big|_0^L \tag{3.27}$$

Using Eq.(3.22) this becomes

$$\frac{K_1}{K_2} n = G-1 \tag{3.28}$$

For any given laser transition frequency, the ratio K_1/K_2 is fixed by the ratio of Einstein A and B coefficients. Thus we say that the overall gain (in excess of trivial unity gain) is proportional to the overall noise and is quite independent of the form of gain function or the density of excited atoms. For the case where the density of the terminal states is not zero or negligible, but $N_2 > N_1$, the gain is reduced by a factor $\frac{N_2 - N_1}{N_2}$, but the noise remains as high.

Therefore $G-1 \leq \frac{K_1}{K_2} n$.

In both cases the necessary signal strength to overcome the noise is as high or higher than was shown in Section 3.1.

2.3 Fluorescence Reduction by Use of Threshold Material

Since one of the great difficulties in image amplification is reducing the background light from fluorescent radiation, consideration has been given to methods for reducing the background light selectively, i.e., without reducing the image output signal. This could be done by finding a material which is saturable, i.e., which has a nonlinear characteristic such as shown by the solid line in Fig. 2.

When the input light intensity is below some threshold very little image contrast can be obtained if the image level is small compared to the fluorescence level. If the threshold is chosen so that it equals the background fluorescent intensity, the amplified signal which radiates on top of this background will have much higher contrast. This material would be placed between the amplifier and the the observer's eye.

A possible threshold material for this purpose is one whose absorption is from the ground state to an excited state and whose relaxation from this excited state is either by radiationless decay or a very fast fluorescent decay time, through an intermediate state. For the threshold material it is necessary to provide a material with the same separation of energy levels, the terminal state being the ground state. The overall operation of this material would be as follows: Light which falls on this material excites its atoms (electrons) in the ground state to the upper level. As increasing light falls on the material, the absorption falls off, (the absorption being due to the difference in the number of atoms between the ground

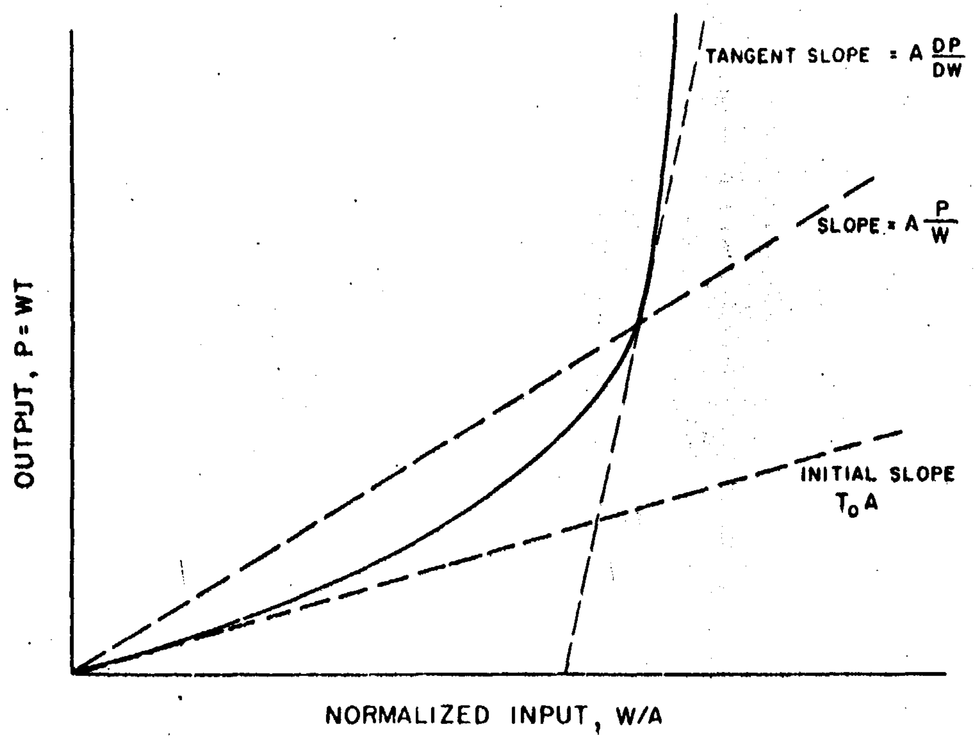


Figure 2 Input-Output Characteristics of Thresholder

state and the excited state). With further increases of light the material becomes almost transparent as the upper and ground states approach equal populations.

If T_o is ratio of input to output at the limit of zero input power

T is ratio of output to input power

W = Input Power

A = Parameter proportional to decay rate of upper state

W/A is measure of power relation to saturation power

$$P_{(x)} = W \exp\left[- \int_{x_1 = 0}^{x_I = x} \alpha dx \right] \tag{3.29}$$

where α is absorption as a function of x and is proportional to the difference of population of ground excited states B_1 and B_2. We may write an equation for the population of excited states

$$\frac{dB_2}{dt} = -K_1 A B_2 + B_1 \alpha_o K_3 P_{(x)} \tag{3.30}$$

where $\alpha_o = \alpha$ when $B_2 = 0$

K_1 = constant

K_3 = constant

Solving for $P_{(x)}$ in the equation and steady state conditions where

$$\frac{dB_2}{dx} = 0$$

and substituting in Eq. (29) we separate variables in P and x. Since $-\log T_o = \alpha_o \ell$, integrating we obtain,

we obtain,

$$-\log T + \frac{W}{A}(1-T) = -\log T_o \tag{3.31}$$

where ℓ = total length

T_o may be taken as a parameter of a specimen with given doping and length, representing the transmission when the input is too small to begin to effect saturation. If WT is plotted as a function of $\frac{W}{A}$ from Eq. (3.31), a curve having the shape of the solid line of Fig. 2 is obtained.

A measure of the benefit obtained from the thresholder is the ratio of the tangential slope $\frac{A/P}{dW}$ to the slope AP/W. On this basis the improvement factor, $F = \frac{dP}{dw} / \frac{P}{W}$ is the quantity to be maximized. In terms of the quantities in the previous equation

$$F = \frac{W/A + 1}{T\ (W/A + 1} \ , \tag{3.32}$$

Alternatively, Eq. (3.31) may be plotted as shown on Fig. 3 showing the transmission T as a function of normalized input W/A, assuming a value of $T_o = 0.05$. An indication of the improvement possible is shown by the curves of Fig. 4 and 5 . In Fig. 4 the improvement factor is plotted as a function of normalized input for various values of T_o. In Fig. 5 the improvement factor is plotted as a function of transmission for various values of T_o.

From Fig. 5 we note that the improvement factor may go as high as 14 if we choose a highly doped long thresholder ($+\log T_o = -20$ or $T_o = 0.2 \times 10^{-8}$) and pump it hard enough to raise its transmission to 0.01. With a sufficient input level such a thresholder provides an improvement factor of 14. Whether the energy from the amplifier

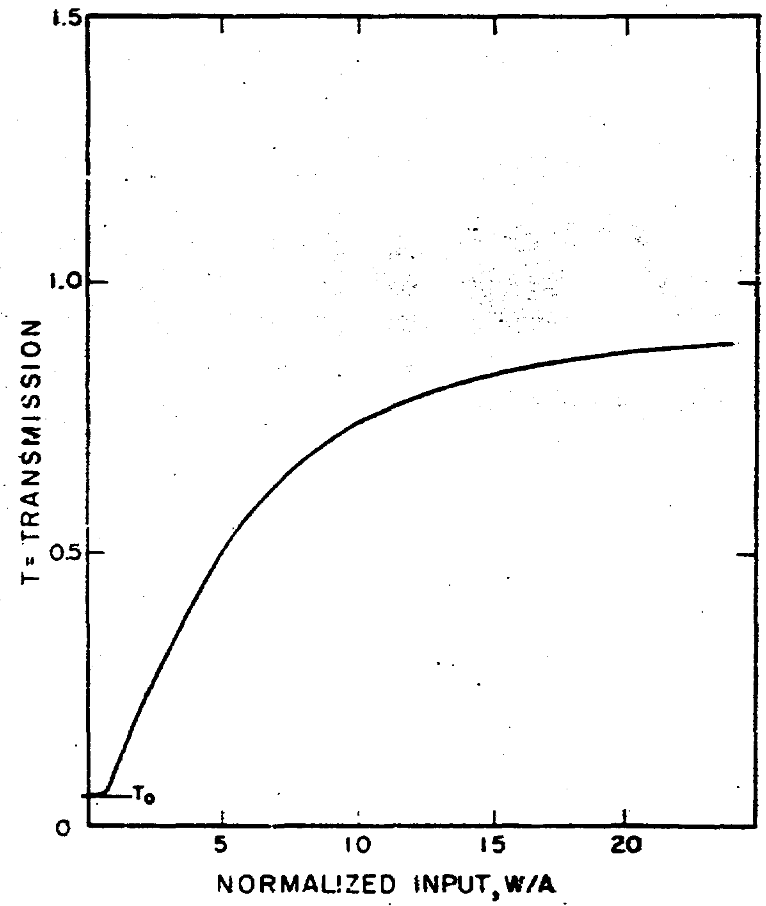


Figure 3 Transmission of Thresholder

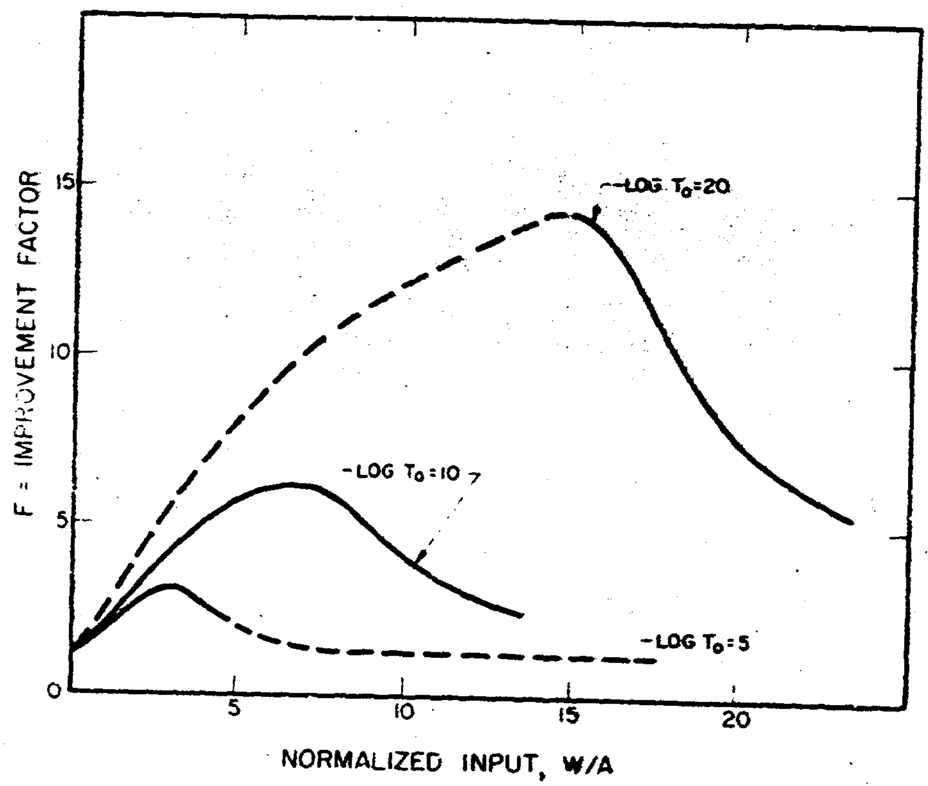


Figure . Improvement Factors Versus Thresholder Input

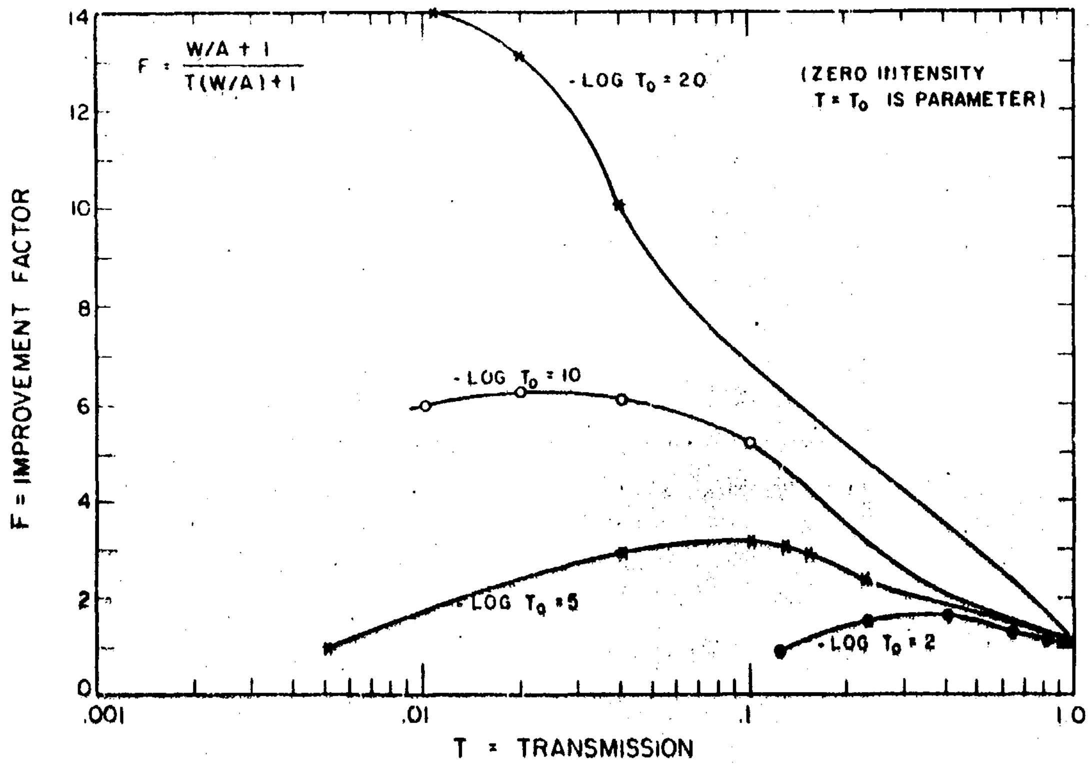


Figure 5 Transmission Versus Improvement Factor of Thresholder

fluorescence is high enough to accomplish such a level of saturation capable of changing absorption from 0.2×10^{-6} to 0.01 as indicated above would require consideration.

In the above discussion reduction of the population of the ground state has been achieved only at the expense of population of the excited state. This excited state population, in turn, gives rise to additional fluorescence. In fact, for every photon absorbed, assuming 100% quantum efficiency, there is one photon of fluorescence. The saving feature is that the fluorescence is omnidirectional whereas the absorbed radiation lies with a certain receiving angle (or field of view). If the field of view were 180° one would only save a factor of 2, i.e., for every two photons entering, only one fluorescence photon falls within the field of view. This gain of 2 will occur at every absorption and reemission. Thus, by either reducing the field of view or increasing the total absorption, one may reduce the effect of fluorescence in the thresholder.

An additional possibility in reducing the thresholder fluorescence is to reduce the quantum efficiency. If the atom decays, either by radiationless decay or radiative decay, to another level with a long decay time, then the ground state will depopulate and the atoms will accumulate in the third state. The fluorescence from and to this third state would be at a different wavelength and hence could be filtered out completely.

The conclusions which can be drawn from the analytical discussions are that substantial contrast gains can be achieved by use of a threshold material but only with large attenuations. Thus, for the thresholder material to work, the amplifier would have to have a large amplification. Since the noise in the amplifier is proportional to the gain and the noise in the thresholder goes down more rapidly than the small signal attenuation, a net signal to noise gain may be achieved.

3. MATERIALS CONSIDERATIONS

At least three criteria are involved in the selection of a stimulated-emission amplifying material for study in imaging work. These are as follows:

(1) Spectral region of amplification
(2) Ratio of fluorescence output power to signal output
(3) Temperature of operation

3.1 Spectral Region of Amplification

Of the dozens of known maser materials, only a few are presently known to be capable of producing amplification in the visible part of the spectrum. A list of these materials, together with their emission wavelength, is shown in Table I.

TABLE I. LIST OF KNOWN LASERS OPERATING IN THE VISIBLE REGION

	MATERIAL	WAVELENGTH IN Å
1. Gas	He-Ne	6328 yellow
2. Semiconductor p-n junction	Gallium Phosphide	~7000
3. Crystals	Sm^{2+} in CaF_2	7082
	Ruby	6943
	Eu^{3+} in Yttrium Oxide	6130
4. Chelates	Europium Benzoylacetonate and Theonyltrifluoroacetonate	6129.5

3.2 Gas Lasers

Although the gas laser has advantages in that it avoids the problems associated with preparation of solid-state crystalline materials and can be relatively easily pumped by direct electrical excitation, its usefulness for the present application appears limited because of the low amplification per unit length in the visible spectrum. Since such lasers have a relatively low density of active material, the maximum density of inverted states is correspondingly low, thus limiting the gain in most cases to about several percent per meter. In order, therefore, to obtain a power gain of 10 times or more, either excessive laser lengths would be involved or more complicated schemes would be required in which the signal radiation is made to pass through the amplifier many times before viewing (causing at the same time a substantial narrowing of the field of view).

3.3 Semiconductor Junction Lasers

The semiconductor junction laser, like the gas laser, has the advantage of being directly excited or pumped by means of electrical input power. In addition, since a high density of carriers can be injected, high gains per unit length can be attained. A further advantage is the fact that the efficiency of conversion of input electrical power to output radiation is relatively high, for example being of the order of 10 percent or more at room temperature for GaAs. Despite the above, however, it is not believed that such types of laser devices are suitable in their present form for purposes of imaging since the active region is limited to the neighborhood of a heavily-doped p-n junction which is of the order of a few microns in thickness.

3.4 Crystal Lasers

As shown in Table I, several types of crystals can produce amplification in the visible spectrum. One of the better known of these is ruby. An important consideration, however, in a stimulated emission amplifier, is the fluorescence level. In the case of ruby, the terminating state of the laser transition is also the ground state of the amplifier which is normally heavily populated. However, in order to obtain amplification, it is necessary that the upper state population be higher than the ground state, requiring that the upper level have a high population density, i.e., greater than that of the ground state. The fluorescence output is thus also very high since the spontaneous emission is proportional to the density of states in the upper level. The above considerations also require that high pumping energies be employed.

Another possible laser crystal is Sm^{2+} in CaF_2. This material has a terminating state for the emission line which is about 263 cm^{-1} in energy above the ground state. Although at room temperature the terminating level is heavily populated, requiring high pump powers for amplification and involving high fluorescence, when the material is cooled sufficiently, for example to liquid helium temperatures, the terminating state becomes essentially empty. In this case the density of states required in the upper level in order to produce population inversion is much smaller than in the case of a two-level system such as ruby. An additional result of cooling the CaF_2 is the fact that the quantum efficiency increases to about 35 percent at liquid helium temperature compared to about 1 percent at room temperature. However, despite its advantages compared to ruby, it is believed that the severe cooling requirement of CaF_2 presents obstacles to its use in intensifier applications.

Another crystalline material of potential use is Eu^{3+} in yttrium oxide. This material, like CaF_2, has a three-level system.

Here the terminating state is about 1000 cm^{-1} above the ground level, the laser transition being $^5F_0 - {}^7F_2$. At room temperature the terminating level is practically empty, being about 1 percent filled with respect to the ground state. Assuming coefficients of stimulated emission and spontaneous emission equal to those for ruby, this material should produce only about 1 percent the background fluorescence output of ruby at the point where population inversion just occurs. At the same time, the pumping power level is correspondingly lower than for ruby. Another advantage of Eu^{3+} in Y_2O_3 is the fact that the quantum efficiency is about 80 percent at room temperature.

3.5 Liquid Organic Lasers

Solutions of europium β-diketone chelates have recently been shown at Electro-Optical Systems, Inc., and other laboratories (Ref. 2, 3, and 4) to sustain laser oscillation at the characteristic strong Eu^{3+} emission wavelength of 6130 Å. These materials are therefore also of interest for light amplification in the visible spectrum. As in the case of Eu^{3+} in yttrium oxide, the terminating state is practically empty so that the ratio of fluorescence-to-signal power is minimized.

One of the basic advantages of the rare-earth chelates is the wide band they possess for the absorption of pump energy. The chelate pump absorption bands are typically 1000 Å wide with peak absorption occurring in the near ultraviolet. The energy absorbed in these bands is quickly and efficiently transferred to the initial or upper laser energy level of the europium ion. Since existing optical pump excitation sources are broad-banded and rich in energy in the absorption region of the chelates, a large fraction of the pump power is available for the production of the required population inversion. Another practical advantage of the chelates is that they may be in liquid form. This avoids the problems associated with crystal preparation and also provides a means for more efficient cooling since the liquid may be circulated.

On the other hand, a number of difficulties arise in the use of chelates. The absorption coefficients for these materials are so high, $\epsilon = 5\times10^4$ liters (mole-cm)$^{-1}$, that the chelate molecule concentration, and therefore the concentration of Eu^{3+} ions, must be low enough to allow the pump radiation to penetrate through the thickness of the laser cell. This means that the rare-earth chelate concentration must be below a few percent for laser cell diameters of several millimeters, and this limits the gain per unit length. In addition, the chemical bond connecting the rare-earth ion to the organic part of the molecule, that is, the rare-earth-to-oxygen bond, is known to be weak, of the order of a few kilocalories per mole. As a result of this, the chelates may be partially decomposed from the pump light, either directly or by local heating.

4. HOMOGENEITY OF POPULATION INVERSION IN A LASER SLAB

The use of an active medium for light amplification in imaging requires that the degree of population inversion be the same throughout the cross section in order to produce a uniforly intensified image. In general, due to the exponential fall of excitation intensity versus the penetration depth, the distribution of the population will be inhomogeneous. (This situation, however, would be different if the excitation intensity is sufficient to saturate the whole medium, but due to obvious difficulties it is not considered here.)

In general, if the absorption coefficient k is small, the exponential term can be expanded as:

$$e^{-kx} = 1 - kx + \text{higher terms}$$

where x is the depth of penetration. For small values of k, the higher terms can be neglected and the fall of intensity then will be approximately linear. Therefore, the smaller the absorption coefficient, the greater is the uniformity of inversion. (The assumption made here is that the inversion is proportional to the intensity of excitation.) Since the absorption coefficient is proportional to the number of laser ions, a medium with minimum density of ions is desirable. On the other hand, the density of inverted population, and hence, the ions should be sufficient so that stimulated emission in them can provide a sufficient net gain.

For the present purpose, an estimate of the uniformity of population inversion will be made for an amplifier of Nd^{3+} doped Barium-Crown glass. The threshold concentration for this material is 0.13 percent.[5] Therefore, 0.2 percent should provide material with substantial gain. Since the absorption coefficient of this material in the green (the main pumping band) is k = 10 for 2 percent doping; for 0.2 percent doping, k can be taken as 1. Assuming a slab of this material, 1 cm thick, optically pumped from both sides, the intensities due to the individual pumps as well as the sum of the intensities as a function of distance from one surface is plotted in Fig. 6b. It is seen that the maximum inhomogeneity is in the region of 10 percent. Fig. 6c shows the same type of plot for a material 0.25 cm thick. The situation for a higher ion concentration (2.3 times more) is shown in Fig. 6a, indicating a non-uniformity of excitation of about 50 percent. If, instead of a slab of material a laser rod of 1 cm diameter is used, the situation will improve somewhat if the pump light is assumed to originate uniformly from all angles around the laser rod, radiating toward the center. In this case, the convergence of the light rays at the center of the rod will help to increase the excitation density at the center. Exact calculations for this purpose involve more complicated mathematics and have not been undertaken in the present study.

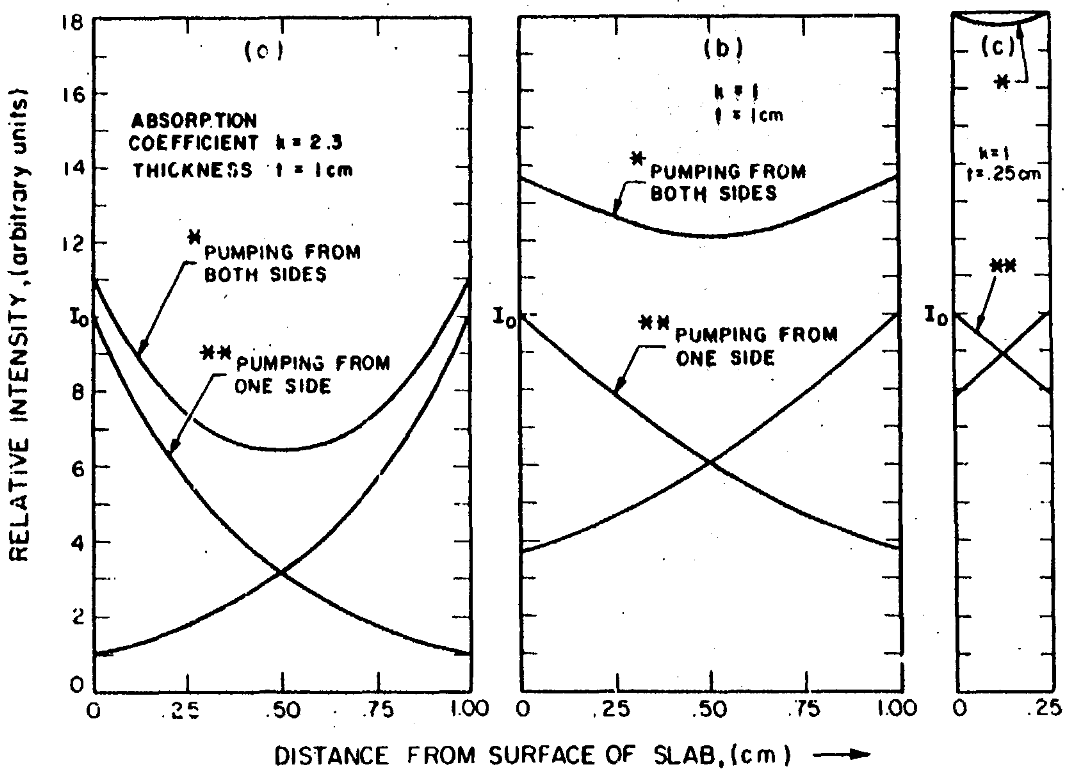


Figure 6 Uniformity of Excitation of a Slab Pumped From Opposite Surfaces

5. IMAGING TESTS*

Aside from the problems of noise, gain, and pumping, an important aspect of the operation of a stimulated emission intensifier is its ability to intensify optical waves without distorting the shape of the wavefronts, which would thus cause loss of resolution. This requirement implies a high degree of uniformity of the optical properties of the amplifying medium and freedom from defects within the material which cause light scattering. Although the full resolution capability of an image intensifier cannot be determined without an operative intensifier of this type, limited tests can be made which will provide preliminary information on some of the problems which might be expected. For this purpose, tests were made as described below on an existing laser amplifier to determine the rectilinear transmission of light rays through it when illuminated by a simple shadow image. Although the tests were made using a neodymium-glass amplifier, operating at a wavelength of 1.06 microns, it is believed that information obtained from this would also be indicative of the problems which might arise with laser rods operating in the visible.

The experimental arrangement is shown diagrammatically in Fig. 7. It consists of a shadow object, a laser oscillator for illuminating the object, a laser amplifier, and a diffuse reflecting screen whose position can be varied. Both the laser oscillator and amplifier are surrounded by the appropriate flash tube, optical pumps, filters, reflectors, etc. The material used for the amplifier is neodymium in glass, fabricated into an eight-inch long, one-fourth inch diameter

* Most of the effort on this phase of the work has been supported by Contract AF 30(602)-2914.

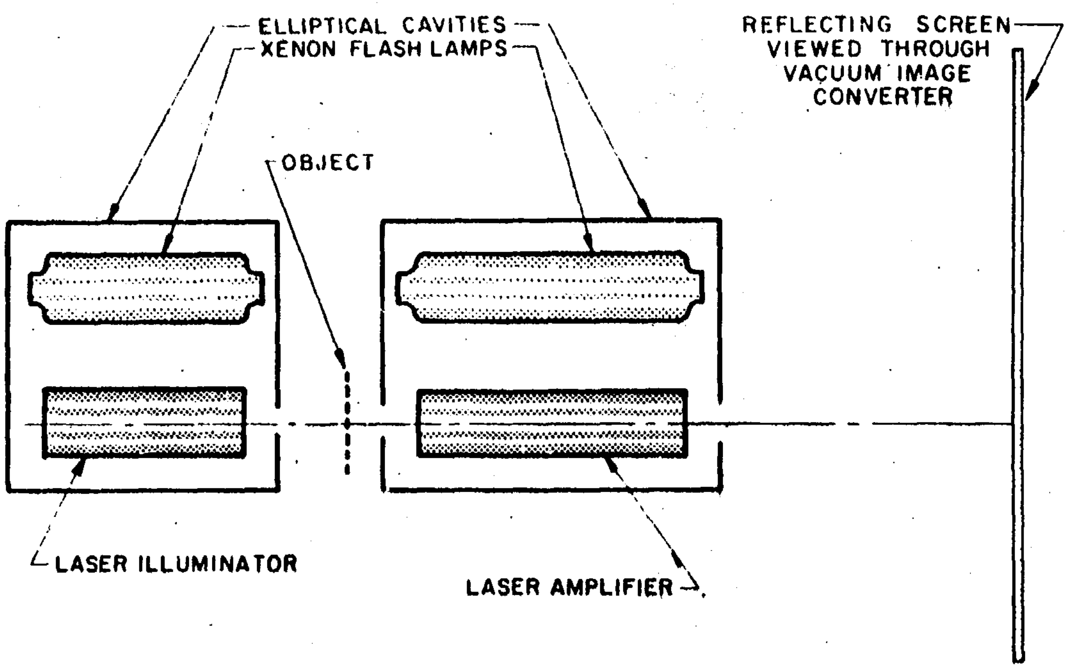


Figure 7 Arrangement Used for Imaging Test

rod. The oscillator consists of a neodymium-doped calcium tungstate crystal rod, one-inch long and one-eighth inch in diameter. For the initial tests, a thin metal wire, about 1 mm in diameter, was used as the shadow object placed about five inches in front of the oscillator.

The experimental procedure was as follows: With the amplifier in position, but unpumped, the oscillator is excited and the shadow of the object is projected through the amplifier onto the reflecting screen. Since the oscillator and amplifier operate at a wavelength of 1.06 microns, the image cannot be observed directly by eye. For viewing purposes, therefore, an image-converter tube was used in the initial experiments. Assuming the oscillator to emit a perfectly plane wave, the image would be projected to infinity with essentially no degradation. However, because of the lack of plane wave generation by the oscillator, a substantial degree of image degradation is produced by this effect alone. The distance beyond the object to the screen where the shadow image becomes so blurred as to become undistinguishable, therefore can be used as a measure of the divergence or lack of parallelism of the oscillator radiation. In other words, the shorter the "blurring distance" the worse the divergence of the radiation.

In the experiments performed, the "blurring distance," with the amplifier unpumped, was 26 ± 1 inches. With both the oscillator and amplifier pumped so that a gain of three is obtained from the amplifier, the "blurring distance" was reduced to 24 ± 1 inches. In view of the fact that considerable stray pumping light and fluorescence light is also present, reducing the image contrast, these two measurements can be considered only slightly different, or essentially equal. The amplifier, when in operation, thus does not appear to seriously affect the rectilinear propagation of light through it.

To obtain more significant results, it is necessary to greatly improve the experimental techniques. For this purpose and also to

obtain a more permanent record of the image, experiments have been conducted with photographic film substituted for the image intensifier arrangement. The only film which has been found to have adequate sensitivity in this spectral region is the Eastman Kodak Type Z, with which further tests are being made.

Another improvement in technique is to use oscillator lasers whose output is more like that of a plane wave. In addition, some degree of improvement can be obtained by stopping the oscillator down in order to use a small portion of its output to limit the divergence. However, one is limited in this regard because of the reduction in light intensity, which causes problems of detection. A still further improvement involves the baffling and filtering out of stray radiation from the pumps and laser fluorescent light.

6. PARAMETRIC AMPLIFICATION AS A POSSIBLE APPROACH TO THE INTENSIFICATION OF OPTICAL IMAGES

6.1 Introduction

As discussed in Section 2, one of the most serious difficulties expected with an image intensifier depending on the use of stimulated emission is the very high background fluorescence light emerging from the amplifier, masking weak signals. Another inherent problem with such amplifiers is their inherently narrow bandwidth. In order to provide a sufficiently high signal level at the input of the amplifier within this narrow bandwidth it is probably necessary to provide narrow band illumination of the scene by means of an auxiliary laser. In view of problems of this type it is essential that other intensification schemes involving optical pumping be investigated to determine if they involve less severe requirements for image intensification. One such type of intensification method depends on the process of parametric amplification.

Parametric amplifiers in the radio frequency and microwave regions of the electromagnetic spectrum are well known to be competitive with MASERS in their very low noise characteristics. Furthermore, they can be designed to have wide bandwidths, in contrast to the MASER which is essentially a narrow banded device. As discussed in another part of this report, optical LASERS present some serious problems in the amplification of weak signals because of their high noise characteristics and also in the amplification of wide banded signals because of their narrow banded characteristics. Thus, it is of basic interest in this research effort to determine for the case of the optical parametric amplifier, both by theoretical analysis and by experiment, the noise, the bandwidth, and the amount of amplification that can be expected,

and also to determine the interrelations that can exist among these parameters.

The application of the principles of the radio frequency and microwave parametric amplifier to the problem of optical amplification has been suggested by Kingston[6], and some of the theoretical treatment has been set forth by Kroll[7]. In addition, Franken and Ward[8] have recently presented a general review of optical harmonics and other nonlinear effects and have included a brief section on the feasibility of the optical parametric amplifier, since its operation is essentially dependent upon these effects. As of the present, however, such amplification in the visible spectrum has not yet been experimentally shown. Nevertheless, it is felt that the possibility definitely exists of developing optical parametric amplifiers utilizing nonlinear effects, and in addition, that parametric oscillators may become sources of coherent electromagnetic radiation at frequencies where direct LASER or MASER action is not feasible.

In the following discussion we present, by means of a consideration of transmission lines in the microwave region, a brief introduction to the principles of the parametric amplifier. The correspondence between the parameters involved in this case and those involved in the optical case is then pointed out, and an explanation of why optical parametric amplification depends upon nonlinear effects in an optical medium is given. Following this, a preliminary concept of an optical parametric amplifier is presented and the possible advantages of low noise and large bandwidths for such a device are discussed.

6.2 Traveling-Wave Parametric Amplification Using Transmission Lines

The basic mechanism of parametric amplification can be simply explained. The device is essentially a variable inductance or capacitance amplifier. As an example, consider a sin wave voltage signal

in a circuit containing a capacitance which can be independently varied with respect to time. At the instant the a.c. voltage reaches a positive maximum, let the capacitance suddenly decrease, say be a sudden increase in the distance between the capacitor plates. When this occurs, work is performed on the circuit and the amplitude of the signal voltage is increased. Subsequently as the voltage becomes zero, let the capacitance go to its original value. This change involves no work on the system and produces no voltage change because the signal voltage was zero when the change was made. Further, when the voltage goes to the minimum negative value, let the capacitance be decreased again. Here again, work is performed on the circuit and the signal voltage amplitude is increased. Finally, when the signal voltage goes to zero, let the capacitance return to its original value. Thus, for one cycle in the signal voltage and two cycles of the proper phase in the variable capacitance, a net increase or amplification of the signal power is obtained at the expense of power expended in changing the capacitor. The capacitor is a power pump, and the frequency of the capacitor variation is the pump frequency.

In the case above, the pump frequency, ω_P, is just twice the signal frequency, ω_S. For the general case it can be shown that $\omega_P = \omega_S + \omega_I$, where power of frequency ω_I, commonly called the idler power, must also be generated in the circuit in order for amplification to occur. For the case above, where $\omega_P = 2\omega_S$ or $\omega_S = \omega_I$, there is no distinction between the signal and idler powers because they are propagated in the same circuit and in the same mode.

The following treatment is presented in order to describe in an analytical fashion the existing theory of the microwave parametric amplifier, and in order to facilitate the recognition of the correspondence between the parameters involved in this case and those involved in the case of the optical parametric amplifier. This treatment parallels closely that given by Tien and Suhl[9], except that they

treat the case where an inductance is the variable parameter whereas we present the case of a variable capacitance.

In Fig. 8 a schematic diagram is given of two lossless transmission lines S and I. These lines are assumed to be loosely coupled by the distributed capacitors $C_p(z,t)$, which vary with time and distance along the line. The latter are varied at the frequency ω_p, assumed to be supplied by a local oscillator or pump. For convenience we divide the transmission lines into small sections and represent each section by a filter circuit. Line S has a phase constant β_S and a characteristic impedance Z_S at an angular frequency ω_S, and line I has a phase constant β_I and a characteristic impedance Z_I at an angular frequency ω_I. We have

$$\beta_S = \omega_S \sqrt{L_S C_S} \quad ; \quad Z_S = \sqrt{\frac{L_S}{C_S}} \tag{7.1}$$

$$\beta_I = \omega_I \sqrt{L_I C_I} \quad ; \quad Z_I = \sqrt{\frac{L_I}{C_I}} \tag{7.2}$$

Line S is excited at the input end by a signal. The function of line I will be understood later: it acts essentially as the idling circuit and for simplification sh[illegible] be open at the input end and terminated at the output end by its characteristic impedance. In the presence of the variable coupling capacitance, the equations of the coupled system are:

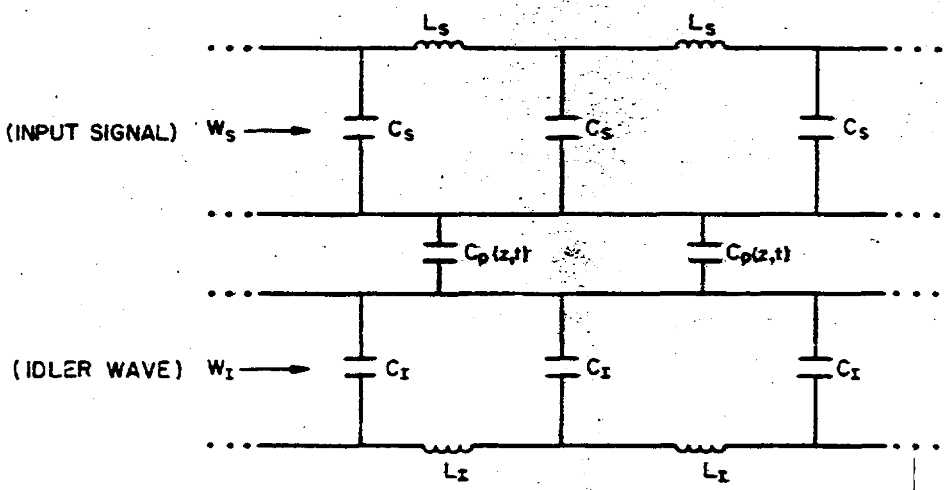


Figure 8 A Capacitively Coupled Traveling-Wave Parametric Amplifier

$$\frac{\partial V_S(z,t)}{\partial z} = -L_S \frac{\partial I_S(z,t)}{\partial t} \tag{7.3}$$

$$\frac{\partial I_S(z,t)}{\partial z} = -C_S \frac{\partial V_S(z,t)}{\partial t} - \frac{\partial}{\partial t} \left[C_p(z,t) V_I(z,t) \right] \tag{7.4}$$

$$\frac{\partial V_I(z,t)}{\partial z} = -L_I \frac{\partial I_I(z,t)}{\partial t} \tag{7.5}$$

$$\frac{\partial I_I(z,t)}{\partial z} = -C_I \frac{\partial V_I(z,t)}{\partial t} - \frac{\partial}{\partial t} \left[C_p(z,t) V_S(z,t) \right] \tag{7.6}$$

Here V_S and I_S are, respectively, the voltage and the current on line S, and V_I and I_I are those on line I. z is the distance along the direction of propagation and t is the time. The terms involving $C_p(z,t)$ are coupling terms or variable parameters responsible for parametric amplification. It is assumed that $C_p(z,t)$ is in the form:

$$C_p(z,t) = \frac{1}{2} \left[C_p(z) e^{j\omega t} + C_p^*(z) e^{-j\omega t} \right] \tag{7.7}$$

$$= \frac{1}{2} C_p \left[e^{j(\omega t - \beta z)} + e^{-j(\omega t - \beta z)} \right] \tag{7.8}$$

when the variable capacitance is modulated by a traveling wave of a phase constant β at an angular frequency ω. Here * denotes the complex conjugate.

We shall only consider waves of the frequency ω_I on line I such that

$$\omega_S + \omega_I = \omega \quad . \tag{7.9}$$

Combining (7.3) and (7.4), and (7.5) and (7.6), we have, respectively,

$$\frac{\partial^2 V_S(z,t)}{\partial z^2} = C_S L_S \frac{\partial^2 V_S(z,t)}{\partial t^2} + L_S \frac{\partial^2}{\partial t^2}\left[C_P(z,t) V_I(z,t)\right] \tag{7.10}$$

$$\frac{\partial^2 V_I(z,t)}{\partial z^2} = C_I L_I \frac{\partial^2 V_I(z,t)}{\partial t^2} + L_I \frac{\partial^2}{\partial t^2}\left[C_P(z,t) \overline{V}_S(z,t)\right] \tag{7.11}$$

Put

$$V_S(z,t) = V_S(z)e^{j\omega_S t} + V_S{}^*(z)e^{-j\omega_S t}$$

$$V_I(z,t) = V_I(z)e^{j\omega_I t} + V_I{}^*(z)e^{-j\omega_I t} \tag{7.12}$$

then (7.10) and (7.11) may be reduced to

$$\frac{\partial^2 V_S(z)}{\partial z^2} = -\omega_S{}^2 L_S C_S V_S(z) - \frac{1}{2}\omega_S{}^2 C_P(z) C_S V_I{}^*(z) \qquad (7.13)$$

$$\frac{\partial^2 V_I{}^*(z)}{\partial z^2} = -\omega_I{}^2 L_I C_I V_I{}^*(z) - \frac{1}{2}\omega_I{}^2 C_P{}^*(z) C_I V_S(z) \qquad (7.14)$$

Similar equations may be obtained for $V_S{}^*(z,t)$ and $V_I(z,t)$ by simply interchanging the subscripts S and I in (7.13) and (7.14).

We shall consider a simple case in which

$$\beta_S + \beta_I = \beta \qquad (7.15)$$

The general case which leads to a more complicated solution will not be treated here. Put

$$V_S(z) = A_S(z) e^{-j\beta_S z}$$

$$V_S{}^*(z) = A_S{}^*(z) e^{j\beta_S z}$$

$$V_I(z) = A_I(z)e^{-j\beta_I z}$$

$$V_I^*(z) = A_I^*(z)e^{j\beta_I z} \tag{7.16}$$

We have also from (7.7) and (7.8),

$$C_p(z) = C_p e^{-j\beta z}$$

$$C_p^*(z) = C_p e^{j\beta z}$$

Denote

$$C_p = \xi_S C_S = \xi_I C_I \tag{7.17}$$

where ξ_S and ξ_I are the ratios of the variable capacitance to the fixed capacitance of line S and Line I, respectively. ξ_S and ξ_I and so the coupling terms in (7.4) and (7.6) are assumed to be small. A(z)'s in (7.16) are then slowly varying functions and the terms involving $\partial^2 A(z)/\partial z^2$'s may be neglected. Substituting (7.16) and (7.17) into (7.13) and (7.14), we have, respectively,

$$-2j\beta_S \frac{\partial A_S(z)}{\partial z} - \beta_S{}^2 A_S(z) = -\omega_S{}^2 L_S C_S A_S(z)$$

$$-\frac{1}{2}\omega_S{}^2 \xi_S L_S C_S A_I{}^*(z) \qquad (7.18)$$

$$2j\beta_I \frac{\partial A_I{}^*(z)}{\partial z} - \beta_I{}^2 A_I{}^*(z) = -\omega_I{}^2 L_I C_I A_I{}^*(z)$$

$$-\frac{1}{2}\omega_I{}^2 \xi_I L_I C_I A_S(z) \qquad (7.19)$$

Since $\beta_S{}^2 = \omega_S{}^2 L_S C_S$ and $\beta_I{}^2 = \omega_I{}^2 L_I C_I$, as given in (7.1) and (7.2), the above equations may be reduced to

$$\frac{\partial A_S(z)}{\partial z} = -\frac{1}{4} j\xi_S \beta_S A_I{}^*(z) \qquad (7.20)$$

$$\frac{\partial A_I{}^*(z)}{\partial z} = \frac{1}{4} j\xi_I \beta_I A_S(z) \qquad (7.21)$$

Combining (7.20) and (7.21), we have

$$\frac{\partial^2 A_S(z)}{\partial z^2} - \frac{1}{16}\xi_S \xi_I \beta_S \beta_I A_S(z) = 0 \qquad (7.22)$$

The solution of (7.22) is

$$A_S(z) = a_1 e^{\alpha z} + b_1 e^{-\alpha z}$$

$$\alpha = \frac{1}{4} (\xi_I \xi_S \beta_I \beta_S)^{1/2} \qquad (7.23)$$

Here a_1 and b_1 are arbitrary complex constants which should be determined by the boundary conditions. From (7.12), (7.16), and (7.23) we have finally

$$V_S(z,t) = e^{\alpha z}\left[a_1 e^{j(\omega_S t - \beta_S z)} + a_1^* e^{-j(\omega_S t - \beta_S z)}\right]$$

$$+ e^{-\alpha z}\left[b_1 e^{j(\omega_S t - \beta_S z)} + b_1^* e^{-j(\omega_S t - \beta_S z)}\right] \qquad (7.24)$$

$$V_I(z,t) = -j\sqrt{\frac{\xi_I \beta_I}{\xi_S \beta_S}}\, e^{\alpha z}\left[a_1^* e^{j(\omega_I t - \beta_I z)} - a_1 e^{-j(\omega_I t - \beta_I z)}\right]$$

$$+ j\sqrt{\frac{\xi_I \beta_I}{\xi_S \beta_S}}\, e^{-\alpha z}\left[b_1^* e^{j(\omega_I t - \beta_I z)} - b_1 e^{-j(\omega_I t - \beta_I z)}\right] \qquad (7.25)$$

Boundary Conditions and General Discussions

It has been shown in (7.24) and (7.25) that for the case $\beta = \beta_S + \beta_I$, both growing and decreasing waves may exist in the coupled transmission-line system. Since the growing wave is always dominant at the output end, the energy has to be transferred from the local oscillator to the growing waves. As mentioned earlier, line S is excited by a signal, and line I is open at the input end. The boundary conditions at the input end $z = 0$ are therefore

$$V_S = a \cos(\omega_S t + \phi)$$

$$V_I = 0 \tag{7.26}$$

Equations (7.24) and (7.25) then become

$$V_S(z,t) = \frac{1}{2} a \left[e^{\alpha z} \cos(\omega_S t - \beta_S z + \phi) + e^{-\alpha z} \cos(\omega_S t - \beta_S z + \phi) \right] \tag{7.27}$$

$$V_I(z,t) = \frac{1}{2} a \sqrt{\frac{\xi_I \beta_I}{\xi_S B_S}} \left[e^{\alpha z} \sin(\omega_I t - \beta_I z - \phi) - e^{-\alpha z} \sin(\omega_I t - \beta_I z - \phi) \right] \tag{7.28}$$

We notice from the above equations that at the input end the growing and the decreasing waves are equal and in phase on line S and are equal and in opposite phases on line I. At a few wavelengths from the input end, the decreasing wave becomes negligible as compared with the growing wave and may be generally ignored in the analysis.

For the general case, $\beta = \beta_S + \beta_I + \Delta\beta$, Tien and Suhl[9] have concluded that the gain is generally reduced when $\Delta\beta$ deviates from zero. We may thus summarize the optimum conditions for amplification as follows:

(1) $$\omega = \omega_S + \omega_I$$

(2) $$\beta = \beta_S + \beta_I$$

(3) $$\left(\frac{d\omega}{d\beta}\right)_S = \left(\frac{d\omega}{d\beta}\right)_I$$

It is obvious that in the propagating circuits, condition (1) is always satisfied. Condition (2) can be easily fulfilled by properly selecting the structures. Condition (3) is necessary in order that condition (2) can hold for a band of frequencies. It indicates that the group velocities of the two lines must be equal in the frequency band of amplification. It may be seen here that for extremely wide bandwidth, we may use the transmission-line type of structures ("TEM" modes) in which the group and the phase velocities are constant for all the frequencies. We may also use helices at the frequencies above their dispersive regions. With these structures, the bandwidth of the amplifier can be very broad, extending from a low frequency up to the energizing frequency.

Power Relations

For simplicity we shall ignore the decreasing waves in (7.27) and (7.28). From (7.27), the power carried by line S is

$$P_S(z) = \left[V_S(z,t)^2/Z_{oS}\right]_{Avg.} = \frac{1}{8} a^2 e^{2\alpha z} \sqrt{\frac{C_S}{L_S}} \tag{7.29}$$

Similarly from (7.28), the power carried by line I is

$$P_I(z) = \left[V_I(z,t)^2/Z_{oI}\right]_{Avg.} = \frac{1}{8} a^2 e^{2\alpha z} \frac{\omega_I}{\omega_S} \sqrt{\frac{C_S}{L_S}} \tag{7.30}$$

The total power transferred from the local oscillator must be the sum of (7.29) and (7.30), which is

$$P_S + P_I = \frac{1}{8} a^2 e^{2\alpha z} \sqrt{\frac{C_S}{L_S}} \left(1 + \frac{\omega_I}{\omega_S}\right) \tag{7.31}$$

Comparing (7.29), (7.30), and (7.31), we find

$$\frac{P_S}{\omega_S} = \frac{P_I}{\omega_I} = \frac{P_S + P_I}{\omega_S + \omega_I} \tag{7.32}$$

This is one of the relations derived by Manley and Rowe[10] in considerations of radio frequency parametric amplifiers. It indicates that the power introduced into the signal line is proportional to the signal frequency, and the higher the signal frequency, relative to the idler frequency ω_I, the higher the amplification.

6.3 Correspondence Between Parameters in the Transmission-Line and Optical Medium

The correspondence between the microwave transmission-line parameters and the optical parametric amplifier parameters can be recognized as follows: By differentiating (7.3) one gets

$$\frac{\partial^2 V_S(z,t)}{\partial z^2} = - L_S \frac{\partial^2 I_S(z,t)}{\partial z \partial t} , \tag{7.33}$$

and by differentiating (7.4) one gets

$$\frac{\partial^2 I_S(z,t)}{\partial t \partial z} = - C_S \frac{\partial^2 V_S(z,t)}{\partial t^2} . \tag{7.34}$$

Substitution of (7.34) into (7.33) then gives the wave equation

$$\frac{\partial^2 V_S(z,t)}{\partial z^2} = L_S C_S \frac{\partial^2 V_S(z,t)}{\partial z^2} \tag{7.35}$$

It is easily shown that for any periodic wave $V_S(z,t)$ which travels at phase velocity υ_S

$$\frac{\partial^2 V_S(z,t)}{\partial z^2} = \frac{1}{\upsilon_S{}^2} \frac{\partial^2 V_S(z,t)}{\partial t^2} , \tag{7.36}$$

so from (7.35) and (7.36)

$$\upsilon_S = \frac{1}{\sqrt{L_S C_S}} . \tag{7.37}$$

The phase velocity of a monochromatic wave in the optical region can be expressed as

$$\upsilon_S = \frac{C}{\eta_S} , \tag{7.38}$$

where C is the velocity of light and η_S is the refractive index of the medium at the frequency ω_S. So from (7.37) and (7.38), $L_S C_S$ in the microwave region corresponds to $\eta_S{}^2/C^2$ in the optical region.

Now the refractive index can be expressed as $\eta_S = \sqrt{\varepsilon\mu}$, where ε is the dielectric constant and μ is the magnetic permeability of a material. Except for ferromagnetic substances which are unimportant

in this discussion, μ is very close to unity, so $\eta_S = \sqrt{\epsilon}$. Also, the dielectric constant depends upon the polarization of the medium as follows:

$$\overline{D} = \epsilon_o \epsilon \overline{E} \tag{7.39}$$

where $\overline{D}$ is the electric flux density, $\overline{E}$ is the electric field intensity, and ϵ_o is the permitivity of free space, a constant dependent on the units employed. But $\overline{D}$ is also given by

$$\overline{D} = \epsilon_o \overline{E} + \overline{P} \quad , \tag{7.40}$$

where $\overline{P}$ is the polarization of the medium. The magnitude of P is commonly expressed as a power expansion in E as follows:

$$P = X_1 E + X_2 E^2 + X_3 E^3 + \ldots \tag{7.41}$$

So from (7.39), (7.40), and (7.41), and the relation $\eta_S = \sqrt{\epsilon}$, one obtains:

$$\eta_S = \left[\left(1 + \frac{X_1}{\epsilon_o} \right) + \frac{X_2}{\epsilon_o} E + \frac{X_3 E^3}{\epsilon_o} + \cdots \right]^{1/2} \tag{7.42}$$

Thus the correspondence between $L_S C_S$ and η_S^2/C^2 is seen to be a correspondence between:

$$L_S C_S \longrightarrow \frac{1}{c^2}\left[\left(1+\frac{X_1}{\epsilon_o}\right)+\frac{X_2 E}{\epsilon_o}+\frac{X_3 E^2}{\epsilon_o}+\cdots\right] \quad ; \qquad (7.43)$$

and a modulation of L_S or C_S, or both, corresponds in the optical region to a modulation of the refractive index of the medium by a modulation of the nonlinear terms $\frac{X_2 E}{\epsilon_o}$, $\frac{X_3 E^2}{\epsilon_o}$, etc. in the polarization. However, these terms are small and their effects can not be readily detected in the optical region when conventional light sources are used as pumps. However, by using a LASER with its high electric field intensities as the source of pump power, the refractive index of a nonlinear medium can be modulated most effectively, so parametric amplification of another light wave in the same medium should be possible.[8]

6.4 Discussion of Image Amplification by Parametric Amplification

In Fig. 9 we indicate an arrangement for image amplification by parametric amplification. An image at frequency ω_S with an unspecified bandwidth is propagated through a nonlinear medium and is subsequently detected by the eye. At the same instance that the image passes through the nonlinear medium, the beam from a high-level monochromatic light source, i.e., a laser is propagated through the same region. This results in a modulation of the dielectric constant of the nonlinear medium and, as a result of this, an amplification of the image energy at ω_S occurs and an idler energy at $\omega_I = \omega_P - \omega_S$ is generated. The image wavefronts are finally detected in intensified form by the eye.

In order to more fully evaluate the above amplification method it is essential that the existing theory describing the pertinent physical phenomena be thoroughly examined and, where the theory is not yet complete, that new theory be developed. Specific problems requiring investigation are the relative orientations of the pump, the signal, and the

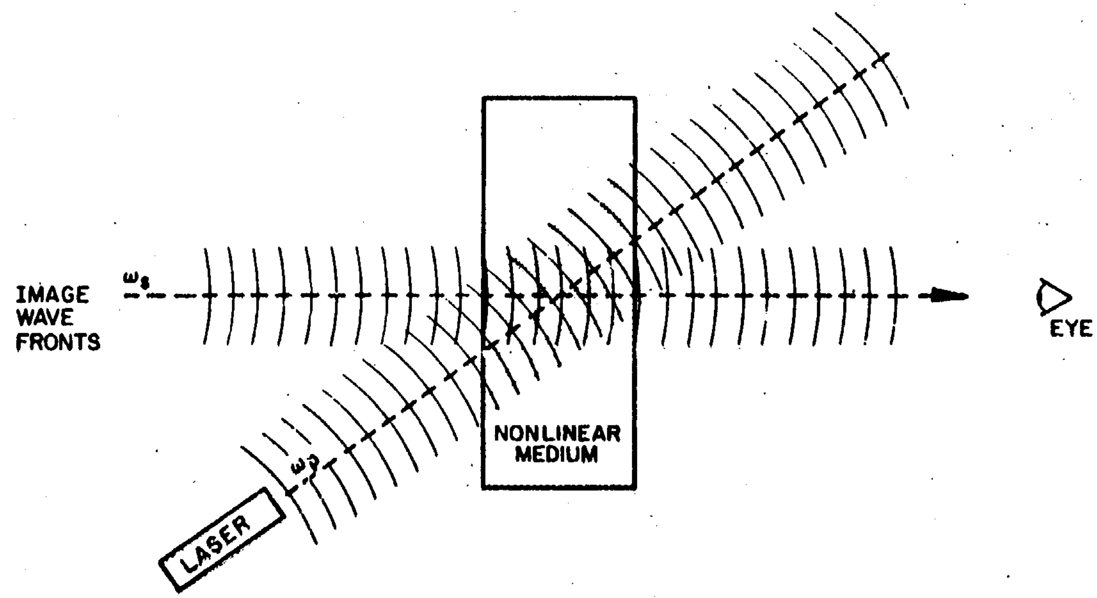


Figure 9 Arrangement for Image Intensification by Parametric Amplification

idler beams; the efficiency of amplification as a function of the relative values of ω_P, ω_S, and ω_I; the amount of noise that might be expected from such a system; the bandwidth, and the amplification as a function of the frequency within the bandwidth; the shape and volume of the amplification region within the material and how this depends upon the other system parameters; and, an important area of study is also the evaluation of the selection of materials that have a suitable nonlinear polarization coefficient.

7. PROGRAM PLANS FOR NEXT QUARTER

7.1 Parametric Amplifier Analysis and Formulation of the Experimental Program

During the next quarter, most of the effort will be directed toward a study of the optical parametric amplification. This study will be essentially analytical with the following areas emphasized because of their importance to problems of optical imaging:

(1) Noise
(2) Bandwidth
(3) Gain
(4) Pump requirements
(5) Nonlinear materials requirements

Quantitative estimates will be obtained for the noise characteristics of the optical parametric amplifier and from these a noise figure comparison will be made with the laser. The bandwidth that can be expected from such a device will be estimated and an analysis will be made to determine how the bandwidth will affect optical images. The gain of the device will be estimated first on the basis of nonlinearities that could be expected from ideal or unknown materials. The pump requirements will be examined in relation to the gain, the image signal frequency, and the optical transmission that can be expected from nonlinear materials. Depending upon the results of the above analysis, experiments will be planned on nonlinear effects and parametric amplification.

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